DEPARTURES FROM SPECIAL RELATIVITY BEYOND EFFECTIVE FIELD THEORIES

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The possibility to have a deviation from relativistic quantum field theory requiring to go beyond effective field theories is discussed. A few recent attempts to go in this direction both at the theoretical and phenomenological levels are briefly reviewed.

1 Introduction

Lorentz invariance is a fundamental ingredient of our present description of Nature, which is given in terms of relativistic quantum field theories (RQFTs). However, the idea that Lorentz invariance might be an approximate, low-energy, symmetry, has begun to emerge in the last few years essentially from quantum gravity developments[1], but also from results in the fields of string theory[2], noncommutative geometry[3], or other ideas such as varying couplings[4].

There are other motivations to think on limitations of the RQFT framework: the strong difficulties in obtaining a RQFT containing gravitation, the large mismatch between the expected and measured value of the energy density of vacuum (cosmological constant problem), or the consideration that in a quantum theory of gravity the maximum entropy of any system should be proportional to the area and not to the volume[5], contrary to what happens in RQFT[6].

There exist also some phenomenological results that could find a simple explanation in terms of a Lorentz invariance violation (LIV). The observation of cosmic rays with energies above 5×10^{19} eV seems to violate the GZK cutoff[7], whose existence is implied by relativistic kinematics. On the other hand, CPT symmetry might also be violated if Lorentz symmetry were not exact, which could provide the key to explain the matter-antimatter asymmetry of our Universe[8].

The framework of effective field theories provides a conservative approach to incorporate departures from special relativity. Here Lorentz symmetry is spontaneously broken, which produces the apparition of Lorentz non-invariant terms in an effective Lagrangian. This approach has been extensively studied in the last years[9]. It assumes that one can incorporate the corrections order by order in the effective theory, so that, for a certain level of precision, the effective Lagrangian has always a finite number of terms. Tests of special relativity have put strong bounds on different terms of the effective Lagrangian[10].

In this paper we want to point out that the effective field theory framework may be too tight to include corrections coming from a LIV, giving explicit examples of theoretical schemes beyond that framework.

2 Beyond effective field theories

2.1 A fundamental theory

With the exception of the attempts to consider a renormalizable theory of gravity at the nonperturbative level[11], all other approaches to a fundamental theory of quantum gravity (QG) go beyond QFT. This includes string theory [12] and their related ideas like a four dimensional theory immersed in a non trivial way in higher dimensions[13]. It is generally assumed that the success of RQFT as a theory of the fundamental interactions of particles (with the exception of gravity) is a consequence of a decoupling between the degrees of freedom which are incorporated in QFT and all the remaining degrees of freedom of the fundamental theory whose virtual effects can be incorporated at the level of QFT. But it might be that this is not the case and some traces remain at low energies which can not be incorporated in a QFT. Our present understanding of the properties of a theory of QG does not allow to exclude this possibility.

The most direct approach would be to look for possible candidates for the fundamental theory and in each case see whether the low energy limit can be described by a QFT. But we are far from the identification of the fundamental theory and a systematic derivation of its low energy limit. It seems then reasonable to explore extensions of QFT as candidates for such low energy limit.

2.2 Non-commutative quantum field theory

A first indication of the possible limitation of a discussion of Lorentz violating effects within the framework of effective field theories comes from the study of noncommutative QFT.

A non-commutativity of space-time has been considered as a possible signature of QG. Then it is natural to consider the formulation of a QFT in a noncommutative space-time[14] as a possible framework for departures from conventional QFT induced by gravitational effects.

A generic feature is the appearance of Lorentz violation effects. One can make explicit computations in different models to identify the quantum effects induced by the non-commutativity. A surprise of this analysis is the appearance of an infrared/ultraviolet (IR/UV) mixing[15], i.e., a dependence of the low energy effective action on the ultraviolet scale M introduced to regulate the UV divergences in loop diagrams. As a consequence of this mixing there is an ambiguity in the estimates of signals of the non-commutativity at low energies. In fact the commutative limit and the low energy limit ($M \to \infty$) do not commute. Depending on how one approaches to the conventional QFT limit one has different results. It may be that the IR/UV mixing has no physical consequences and the effects of a noncommutative space-time can be incorporated at the level of effective field theories[3] or alternatively one can have a remaining non-locality due to extended degrees of freedom at low energies which can not be incorporated in the effective field theory framework[16]. Which of these cases (if any) is realized in QG will depend on the details of the underlying theory.

2.3 Quantum theory of non-commutative fields

Another way to go beyond conventional QFT is based on the introduction of a non-commutativity in the space of field configurations [17]. The canonical commutation relations between the field and the conjugated momentum at each space point is supplemented by a non trivial commutator between different fields and/or between different conjugated momenta. When these generalized commutation relations are combined with the conventional field theory Hamiltonian one has a generalization of QFT with a violation of Lorentz invariance.

The simplest model with non-commutative fields [17] is the free theory of two scalar fields. There are two different type of excitations (the particle-antiparticle symmetry is broken by the non-commutativity of the fields). Two energy scales parametrize the generalization of the commutation relations. When the ratio of these two energy scales is very small there is a domain of energies between the two scales where one approaches the conventional theory of relativistic particles and antiparticles.

In order to illustrate the dynamical consequences of the non-commutativity of fields one can consider the Hamiltonian formulation of non-commutative scalar QED. One has a Hamiltonian

$$H = H_g + H_m \tag{1}$$

where

$$H_g = \frac{1}{2}\mathbf{E}^2 + \frac{1}{2}\left(\nabla \wedge \mathbf{A}\right)^2 \tag{2}$$

is the Hamiltonian of the electromagnetic field with commutators

$$[A_j(\mathbf{x}), E_k(\mathbf{y})] = i\delta_{jk}\delta\left(\mathbf{x} - \mathbf{y}\right) \tag{3}$$

and

$$H_m = \Pi^\dagger \Pi + \left(\nabla\Phi^\dagger + i\mathbf{A}\Phi^\dagger\right)\left(\nabla\Phi - i\mathbf{A}\Phi\right) + m^2\Phi^\dagger\Phi \tag{4}$$

is the Hamiltonian of the matter system corresponding to the theory of a complex non-commutative scalar field with commutators

$$\left[\Phi(\mathbf{x}), \Pi^\dagger(\mathbf{y})\right] = -\left[\Pi(\mathbf{x}), \Phi^\dagger(\mathbf{y})\right] = i\delta\left(\mathbf{x} - \mathbf{y}\right) \tag{5}$$

$$\left[\Phi(\mathbf{x}), \Phi^\dagger(\mathbf{y})\right] = \frac{1}{\Lambda}\delta\left(\mathbf{x} - \mathbf{y}\right) \tag{6}$$

$$\left[\Pi(\mathbf{x}), \Pi^\dagger(\mathbf{y})\right] = \lambda\,\delta\left(\mathbf{x} - \mathbf{y}\right) \tag{7}$$

Physical states satisfy the constraint

$$[\nabla\mathbf{E}(\mathbf{x}) - \rho(\mathbf{x})]\left|\Psi\right\rangle_{phys} = 0 \tag{8}$$

with

$$Q = \int d^3\mathbf{x}\,\rho(\mathbf{x}) \tag{9}$$

the generator of $U(1)$ transformations of the scalar field.

It is straightforward to go from the Hamiltonian to the Lagrangian formulation following step by step the case of conventional scalar QED. The final result is a Lagrangian

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m \tag{10}$$

with

$$\mathcal{L}_g = \frac{1}{2}\left(\partial_t \mathbf{A} - \nabla A_0\right)\left(\partial_t \mathbf{A} - \nabla A_0\right) - \frac{1}{2}\left(\nabla \wedge \mathbf{A}\right)\left(\nabla \wedge \mathbf{A}\right) \tag{11}$$

which is the conventional Lagrangian of the electromagnetic field (non-commutativity is introduced in the matter sector). The modified Lagrangian of the matter system is given by

$$
\begin{aligned}
\mathcal{L}_m =\ & -\left(\frac{1}{1 - \lambda/\Lambda}\right)^2 \Phi^\dagger \frac{\left(\partial_t - iA_0\right)^2}{1 - i\left(\partial_t - iA_0\right)/\left(\Lambda - \lambda\right)} \Phi \\
& + \Phi^\dagger \left(\nabla - i\mathbf{A}\right)^2 \Phi - m^2 \Phi^\dagger \Phi
\end{aligned}
\tag{12}
$$

One can see that the non-commutativity translates into a non-locality in the Lagrangian which makes manifest how the theory of non-commutative fields goes beyond the effective field theory approach.

2.4 New infrared scales

Effective field theories have a limited range of applicability. They are supposed to give sensible descriptions at energies E much lower than a high-energy scale M, the ultraviolet cutoff of the theory, whose effect can be incorporated by nonrenormalizable terms which produce corrections of order (E/M). The scale M corresponds usually to the mass of a very massive particle, so that the introduction of this energy scale does not pose any problems with respect to relativistic invariance.

On the other hand, the introduction of corrections to a quantum field theory parametrized by a low-energy scale has not been so well explored in the literature. In fact, depending on how this energy scale is introduced, these corrections could violate relativistic invariance. There are, however, several phenomenological and theoretical reasons that lead to think on the necessity to incorporate a new IR scale to our theories. These include:

1. The seeming existence of a cosmological constant or a vacuum energy density whose experimental value, $\rho_V \sim (10^{-3}\,\mathrm{eV})^4$ is 124 orders of magnitude lower than its expected value from ordinary quantum field theory [18].

2. The fact that the entropy in a field theory scales with the volume, while in a quantum theory of gravity the maximum entropy should be proportional to the area leads to think that conventional quantum field theories overcount degrees of freedom [5], which suggests the breakdown of any effective theory with an ultraviolet cutoff to describe systems which exceed a certain critical size which depends on the ultraviolet cutoff [6]. This critical size constitutes an IR energy scale.

3. In the approach of large extra dimensions [19], the observed hierarchy between the electroweak and Planck scales is explained by postulating a fundamental scale $M \sim 10 - 100$ TeV of gravity along with Kaluza-Klein compactification with large radius R, so that the Planck scale is then an effective four-dimensional scale. The inverse of R is an IR energy scale.

4

A deviation from a RQFT at low energies due to an IR scale λ may well be expected to violate relativistic invariance. The reason is that it has been shown[20] that *any* theory incorporating quantum mechanics and special relativity, with an additional "cluster" condition[20], must reduce to a RQFT at low energies.

A simple way of incorporating effects beyond RQFT that violate relativistic invariance is through a modified dispersion relation. For example, the dispersion relation

$$E^2 = \boldsymbol{p}^2 + m^2 + \lambda|\boldsymbol{p}|, \tag{13}$$

has a term linear in $|\boldsymbol{p}|$ which dominates over the standard kinetic term $(\boldsymbol{p}^2)$ when $|\boldsymbol{p}| \lesssim 2\lambda$ and so it changes drastically the nonrelativistic kinematics. For instance, such a dispersion relation for the electron would slightly modify the energy levels of the hydrogen atom. Given the extraordinary agreement between theory and the experimental measurement of the Lamb shift (one part in 10^5 [20,21]), one has $\lambda < 10^{-6} - 10^{-7}$ eV, which is a very stringent bound on the IR scale.

The bound is much less restrictive and more interesting if one considers that Eq. (13) applies to the neutrino only because of a special sensitivity of this particle to the IR scale. This idea could find theoretical support in the framework of large extra dimensions, considering that the neutrino is the only particle, together with the graviton, which propagates in the extra dimensions, and that the dependence on the scale λ is suppressed for the remaining particles, which would then explain why no signal of these LIVs has been observed. The neutrino has two characteristic properties: it has a very small mass, and it interacts only weakly. As a result of this combination, we have not any experimental result on its nonrelativistic physics. Therefore, the presence of Lorentz invariance violations affecting the nonrelativistic limit cannot be excluded a priori in the neutrino case.

In fact Eq. (13) for the neutrino has been used[22] to explain the tritium beta-decay anomaly, which consists in an excess of electron events at the end of the spectrum, at about 20 eV below the end point[23]. Matching with experimental results requires a value of λ in Eq. (13) of the order of the eV. It can be seen[22] that this does not contradict other experimental results involving neutrinos, such as their contribution to the energy density of the Universe, neutrino oscillations (if the IR scale is family-independent) or neutrinos from supernovas, although this could be a good place to look for footprints of this LIV.

Eq. (13) for the neutrino might however have important effects in cosmic rays through threshold effects which become relevant when $\lambda|\boldsymbol{p}_{\text{th}}| \sim m^2$. Here m^2 is an "effective" mass squared which controls the kinematic condition of allowance or prohibition of an specific process. Indeed a consequence of these threshold effects could be that neutrons and pions become stable particles at energies close to the knee of the cosmic ray spectrum[22], which would drastically alter the composition of cosmic rays. It is quite remarkable that cosmic ray phenomenology could be sensitive to the presence of an IR scale.

A modified dispersion relation of the form of Eq. (13) cannot be simply introduced in the framework of an effective field theory, because the term proportional to $|\boldsymbol{p}|$ cannot be Taylor-expanded. We therefore lack of a dynamical formalism consistent with such a dispersion relation, and we can only explore for the moment its kinematic implications.

There is another difficulty of introducing a LIV through a modified dispersion relation, that is, one should indicate the "preferred" frame in which this relation is valid. There is however another possibility, which is to extend our concept of relativistic invariance

to a more general framework, in which the new dispersion relation would be observer-invariant. This case is considered in the following section.

2.5 Double Special Relativity

The possibility that Lorentz symmetry, considered as a low-energy symmetry that would not be exactly preserved in a quantum theory of gravity, might not be broken in the fundamental theory, but only deformed to a different symmetry, was started to be explored quite recently[24]. This deformation usually involves[25] a new dispersion relation of the form $m^2 = f(E, p; M)$, with $f(E, p; M) \to E^2 - p^2$ in the $M \to \infty$ limit, where M is a large-energy/small-length (possibly related to the Planck mass) scale, which is introduced as an observer-independent scale. The new dispersion relation then implies new laws of boost/rotation transformation between inertial observers. As Galilean Relativity was deformed to Special Relativity in order to introduce a relativistic-invariant scale (the speed of light), the new framework [therefore called *Doubly Special Relativity* (DSR)] deforms Special Relativity to introduce this second observer-independent scale.

There is an special point that needs to be remarked in the kinematical analysis of physical processes in the DSR framework. The assumption of modified dispersion relations and unmodified laws of energy-momentum conservation is inconsistent with the doubly-special relativity principles, since it inevitably[24] gives rise to a preferred class of inertial observers. A doubly-special relativity scenario with modified dispersion relations must therefore necessarily have a modified law of energy-momentum conservation.

On the contrary, unmodified laws of energy-momentum conservation are an ingredient of any low-energy effective field theory. This fact is an indication that DSR theories are not included in the effective field theory framework.

It is interesting to note that one can find a phenomenologically consistent *infrared* DSR [18], meaning the introduction of a new IR scale λ, as discussed in the previous section, but in a relativistic-invariant way. In particular, it is possible to obtain appropriate deformed Lorentz transformations such that the following dispersion relation

$$\tilde{m}^2 = \left(1 - \frac{\lambda}{E}\right)^2 (E^2 - \boldsymbol{p}^2), \tag{14}$$

remains invariant. In the nonrelativistic limit, this dispersion relation is reduced to

$$E \simeq m + \frac{\boldsymbol{p}^2}{2m} - \frac{\tilde{m}}{m} \frac{\boldsymbol{p}^4}{8m^3}, \tag{15}$$

where we have defined the physical mass parameter $m = \tilde{m} + \lambda$. Tests of QED now give a bound for the IR scale $\lambda \lesssim 10^{-2}$ eV [18]. The kinematical dependence on the IR scale in the case of DSR is very different from the one considered in the previous section. This is the reason why the bounds on the IR scale from precision QED tests differ by several orders of magnitude. Interestingly enough, a value of the new scale of order 10^{-3} eV could be detected in the near future from its QED effects, and, at the same time, provide a solution to the cosmological constant problem. Assuming a mechanism of cancelation for the different contributions to the vacuum expectation value of the energy momentum

tensor, then on dimensional grounds one would expect

$$\langle T_{\mu\nu} \rangle \sim \lambda^4 \left(c_1 \delta_\mu^0 \delta_\nu^0 + c_2 \sum_i \delta_\mu^i \delta_\nu^i \right) \tag{16}$$

with c_1, c_2 dimensionless coefficients depending on the details of the theory incorporating the new low energy scale. Then the present acceleration of the expansion of the Universe could be a signal of a new low energy scale compatible with a modified relativity principle.

3 Conclusions

We are entering in a period where an experimental search of QG effects is possible. One of the most clean signals of these effects would be a departure from special relativity, since ultra-high energy cosmic rays and astrophysical observations can provide amplification mechanisms (through kinematical thresholds, modification of stability or instability conditions for high-energy particles, or time-of-flight measurements of far enough energy sources) by which these effects might be observable in a close future. On the other hand, QED and/or neutrino physics could also detect the presence of an IR scale producing a slight modification of the kinematics in the nonrelativistic limit.

The details of the way one approaches to a special relativistic theory will give us very important hints on the underlying theory. Lacking the basic principles of this theory one should keep an open mind and explore different alternatives for extensions of RQFT considering also the possibility to go beyond effective field theories. We have sketched a few of these alternatives as candidates for a phenomenological perspective to the analysis of QG effects.

Acknowledgments

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